



HKU USP – NCS Primary Mathematics 2017–19

Division Algorithm: Students' Difficulties and Teaching Ideas

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Students' Difficulties

On Division Algorithm:

P.2 ✓ P.3 ✗

$31 \div 7 = \boxed{4} \dots \boxed{3} \quad |$
 $72 \div 6 = \underline{11 \dots 6} \quad | \quad 0$

$31 \div 7 = \boxed{4} \dots \boxed{3} \quad |$
 $72 \div 6 = \underline{9 \dots 18} \quad | \quad 0$

$31 \div 7 = \boxed{4} \dots \boxed{3} \quad |$
 $72 \div 6 = \underline{6 \dots 12} \quad | \quad 0$

$= \boxed{4} \dots \boxed{3} \quad |$
 $= \underline{102} \quad | \quad 0$

$31 \div 7 = \boxed{4} \dots \boxed{3}$
 $72 \div 6 = \underline{21}$

$1 \quad 7 \overline{) 28} \quad 4$
 $\quad \underline{28}$
 $\quad \quad 0$
 $\quad \quad \quad 3$

$6 \overline{) 72} \quad 21$
 $\quad \underline{6}$
 $\quad \quad 12$
 $\quad \quad \underline{12}$
 $\quad \quad \quad 0$

$7 \overline{) 51} \quad 4$
 $\quad \underline{28}$
 $\quad \quad 3$

$6 \overline{) 72} \quad 102$
 $\quad \underline{60}$
 $\quad \quad 12$
 $\quad \quad \underline{12}$
 $\quad \quad \quad 0$

On Division Algorithm:

P.2 ✗ P.3 ✓

$$7 \mid 31 \div 7 = \boxed{3} \square \dots \boxed{7} \times$$
$$72 \div 6 = \underline{12} \checkmark$$

$$\begin{array}{r} 12 \\ 6 \overline{) 72} \\ \underline{6} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

$$\begin{array}{r} 31 \\ 7 \overline{) 31} \\ \underline{21} \\ 10 \\ \underline{7} \\ 3 \end{array}$$

$$7 \mid 31 \div 7 = \boxed{31} \dots \boxed{77} \times$$
$$72 \div 6 = \underline{12} \checkmark$$

$$\begin{array}{r} 12 \\ 6 \overline{) 72} \\ \underline{6} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

$$\begin{array}{r} 31 \\ 7 \overline{) 31} \\ \underline{21} \\ 11 \\ \underline{7} \\ 4 \end{array}$$

On Division Algorithm:

P.2 x P.3 ✓

7	$31 \div 7 = \boxed{4} \dots \boxed{2} \quad 0$ $72 \div 6 = \underline{12} \quad 1$	$\begin{array}{r} 40 \\ 7 \overline{) 31} \\ \underline{28} \\ 2 \\ \underline{0} \\ 2 \end{array}$
8	Share a bar of chocolate equally among 5 children.	

7	$31 \div 7 = \boxed{4} \dots \boxed{17} \quad \times$	$\begin{array}{r} 4 \\ 7 \overline{) 31} \\ \underline{28} \\ 17 \end{array}$	$\begin{array}{r} 12 \\ 6 \overline{) 72} \\ \underline{6} \\ 12 \\ \underline{12} \end{array}$
	$72 \div 6 = \underline{12} \quad \checkmark$		

On Division Algorithm:

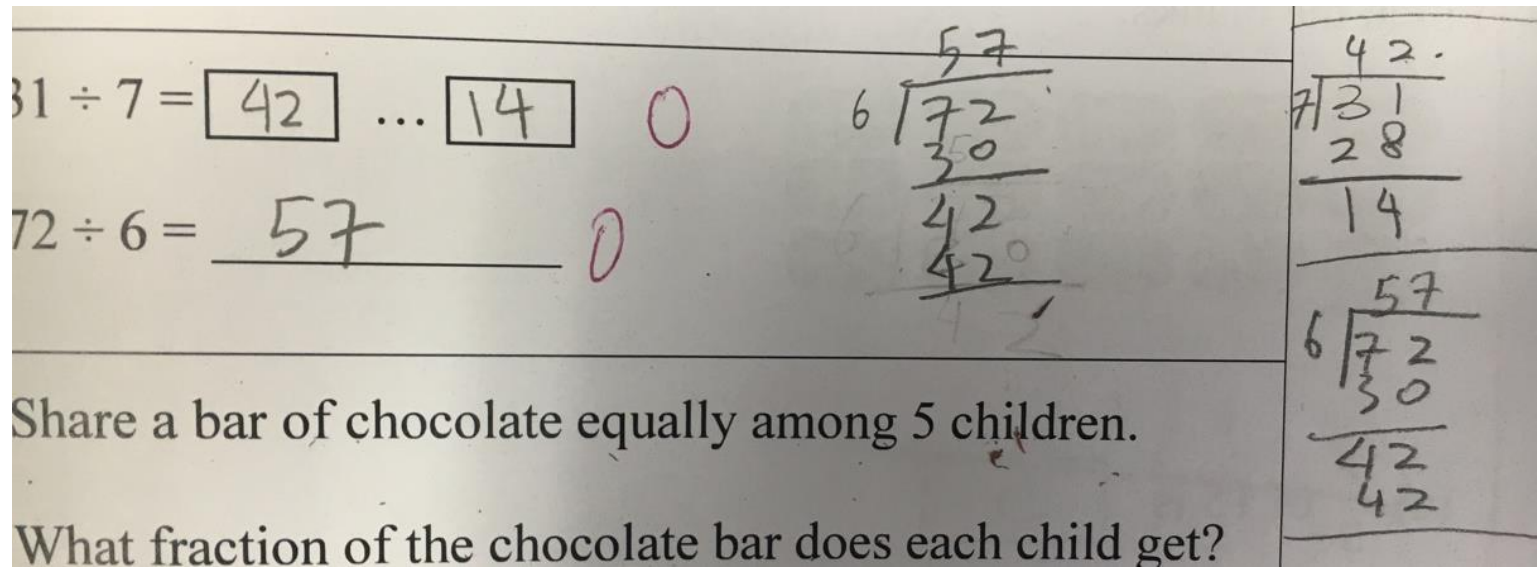
P.2 x P.3 x

$31 \div 7 = \boxed{4} \dots \boxed{4} \quad 0$

$72 \div 6 = \underline{12} \quad 0$

Share a bar of chocolate equally among 5 children.

What fraction of the chocolate bar does each child get?



The image shows a piece of lined paper with handwritten mathematical work. On the left, there are two division problems: $31 \div 7 = \boxed{4} \dots \boxed{4} \quad 0$ and $72 \div 6 = \underline{12} \quad 0$. The boxes and underlines are drawn by hand. To the right of these are two long division problems. The first is $6 \overline{)72}$ with a quotient of 12 and a remainder of 0. The second is $7 \overline{)31}$ with a quotient of 4 and a remainder of 4. Below the word problem, there is a blank space for the answer.

On Division Algorithm:

P.2 ✓ P.3 ✓

$$7 \overline{) 31} \div 7 = \boxed{4} \dots \boxed{3} \checkmark$$
$$72 \div 6 = \underline{12} \checkmark$$

$$\begin{array}{r} 41 \\ \hline 31 \\ 28 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 12 \\ \hline 6 \overline{) 72} \\ 6 \\ \hline 12 \\ 12 \\ \hline \end{array}$$

Students' Difficulties on Division Algorithm

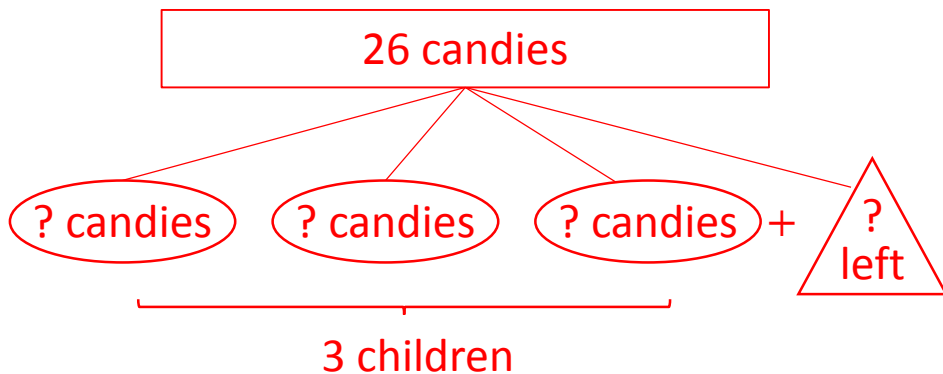
- Unable to extend the algorithm to the case $\text{dividend} > 9 \times \text{divisor}$
- Do not understand the meaning of the place values of the digits of the quotient

Teaching Ideas

P.2: Basic Division

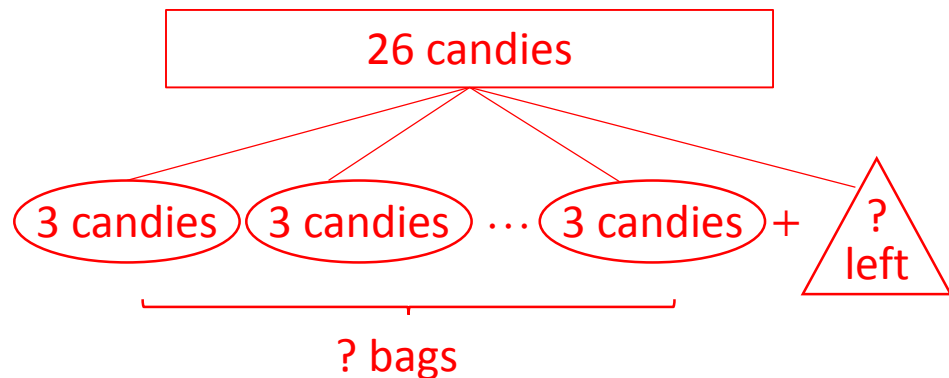
Sharing, Grouping and Division

Share 26 candies equally among 3 children.



$$\begin{array}{r} ? \\ 3 \overline{) 26} \\ \underline{\quad ?} \\ ? \end{array}$$

26 candies, put 3 candies into each bag.

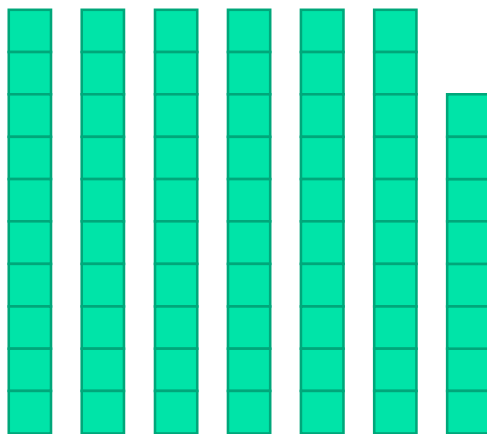


$$\begin{array}{r} ? \\ 3 \overline{) 26} \\ \underline{\quad ?} \\ ? \end{array}$$

P.3: Dividing 2 or 3 digits numbers by 1 digit number

www.mathlearningcenter.org/web-apps/number-pieces

e.g. $68 \div 3 = ?$

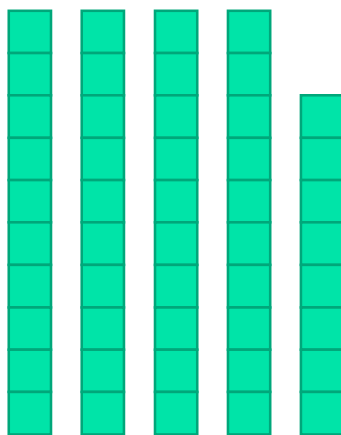


	T	U
	2	2
3	6	8
	6	0
		8
		6
		2

P.3: Dividing 2 or 3 digits numbers by 1 digit number

www.mathlearningcenter.org/web-apps/number-pieces

e.g. $48 \div 3 = ?$

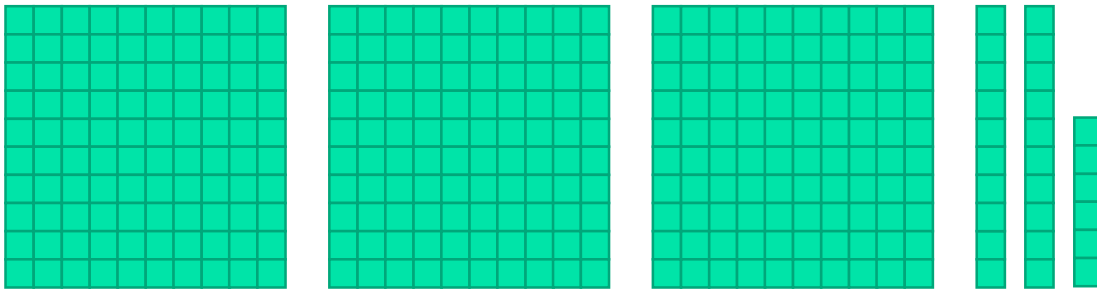


	T	U
	1	6
3	4	8
	3	0
	1	8
	1	8

P.3: Dividing 2 or 3 digits numbers by 1 digit number

www.mathlearningcenter.org/web-apps/number-pieces

e.g. $326 \div 3 = ?$



	H	T	U
	1	0	8
3	3	2	6
	3	0	0
		2	6
		2	4
			2

Alternative Ways?

		1
		3
7)	31
		21
		10
		7
		3

} 4

		4
		10
6)	86
		60
		26
		24
		2

} 14

		5
		9
6)	86
		54
		32
		30
		2

} 14

			8	
		1	0	0
4)	4	3	5
		4	0	0
			3	5
			3	2
				3

} 108

疊除法

Final Remarks

