EXERCISE counts!

PRACTICE MAKES PERFECT!?

Designing exercise that promotes Conceptual understanding & Self-regulated learning

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September 27, 2019

Practice makes perfect!?

typical exercise

練習十九　　　　日期：______

計算下列各題。

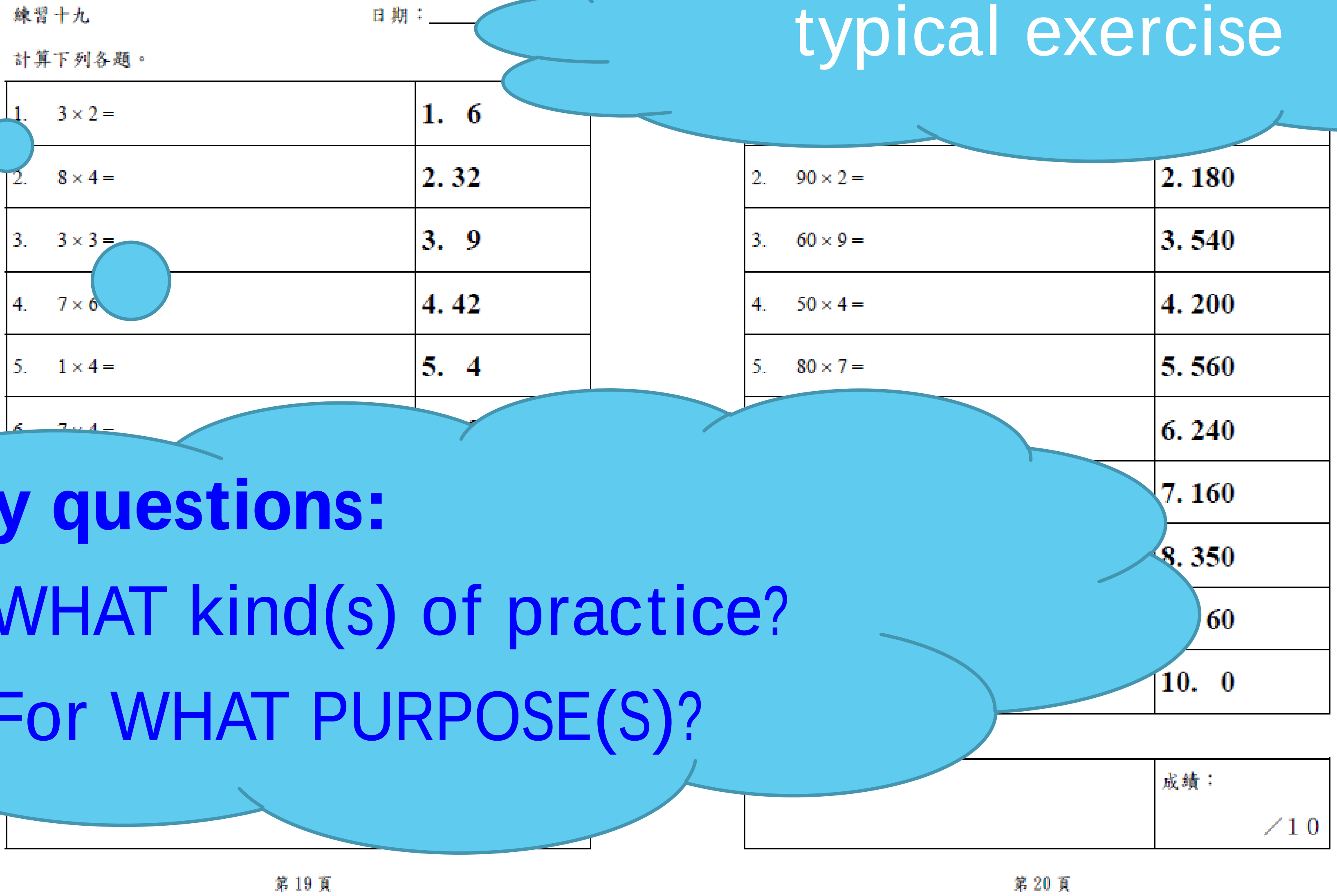
1.　3 × 2 =	1.　6
2.　8 × 4 =	2. 32
3.　3 × 3 =	3.　9
4.　7 × 6 =	4. 42
5.　1 × 4 =	5.　4
6.　7 × 4 =	6. 28
7.　6 × 9 =	7. 54
8.　2 × 5 =	8. 10
9.　8 × 2 =	9. 16
10.　9 × 7 =	10.　63

學習日誌：	成績： ／10

2.　90 × 2 =	2. 180
3.　60 × 9 =	3. 540
4.　50 × 4 =	4. 200
5.　80 × 7 =	5. 560
6.　40 × 6 =	6. 240
7.　20 × 8 =	7. 160
8.　70 × 5 =	8. 350
9.　30 × 2 =	9.　60
10.　60 × 0 =	10.　0

學習日誌：	成績： ／10

Practice makes perfect!?



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第 19 頁

2. 90 × 2 =	2. 180
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	6. 240
	7. 160
	8. 350
	60
	10. 0
	成績：／10

第 20 頁

typical exercise

Key questions:

- WHAT kind(s) of practice?
- For WHAT PURPOSE(S)?

2 × 3 =	6 × 7 =	9 × 8 =
2 × 30 =	6 × 70 =	9 × 80 =
2 × 300 =	6 × 700 =	9 × 800 =
20 × 3 =	60 × 7 =	90 × 8 =
200 × 3 =	600 × 7 =	900 × 8 =

Shanghai Textbook Grade 2 (aged 7/8)

Key questions:

- WHAT kind(s) of practice?
- For WHAT PURPOSE(S)?

Example: Practice with multiplication and …

1. 19 x 2 =
2. 18 x 3 =
3. 17 x 4 =
4. 16 x 5 =
5. 15 x 6 =
6. 14 x 7 =
7. 13 x 8 =
8. 12 x 9 =

練習十九　　日期：__________

計算下列各題。

1. 3 × 2 =	1. 6
2. 8 × 4 =	2. 32
3. 3 × 3 =	3. 9
4. 7 × 6 =	4. 42
5. 1 × 4 =	5. 4
6. 7 × 4 =	6. 28
7. 6 × 9 =	7. 54
8. 2 × 5 =	8. 10
9. 8 × 2 =	9. 16
10. 9 × 7 =	10. 63

練習二十　　日期：__________

計算下列各題。

1. 10 × 9 =	1. 90
2. 90 × 2 =	2. 180
3. 60 × 9 =	3. 540
4. 50 × 4 =	4. 200
5. 80 × 7 =	5. 560
6. 40 × 6 =	6. 240
7. 20 × 8 =	7. 160
8. 70 × 5 =	8. 350
9. 30 × 2 =	9. 60
10. 60 × 0 =	10. 0

成績：

Can you write out another set of multiplication expressions like the ones you have just done?

Example: Simple addition and …

算一算，照样子分别再写出一组算式。

$3 \times 7 =$	$63 \div 9 =$
$30 \times 7 =$	$630 \div 9 =$
$300 \times 7 =$	$6300 \div 9 =$

Mathematics Book 3A published by Beijing Normal University 2014 (p. 38)

Example: Practice with multiplication and division

□里应该填几?

□ × 2 = 60	□ × 2 = 24	□ × 3 = 60	□ × 3 = 12
□ ÷ 2 = 60	□ ÷ 2 = 24	□ ÷ 3 = 60	□ ÷ 3 = 12

Can you write out another set of expressions like the ones you have just done?

Mathematics Book 3A published by Beijing Normal University 2014 (p. 38)

Key Idea: Patterns and systematic variation

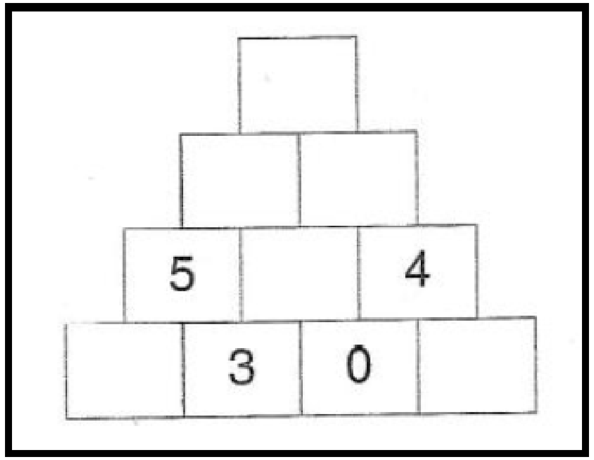
Even more different kinds of practices / exercises in the following slides may help to illustrate the main ideas and considerations for the purpose.

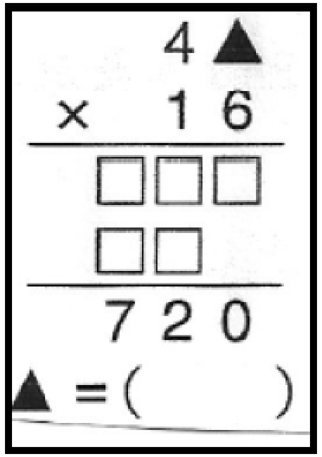
These exercises provide both the opportunity to practice calculation whilst at the same time thinking about the relationships within the mathematics.

Retreived February 21, 2018 at https://www.ncetm.org.uk/public/files/22344481/Variation%20supports%20Intelligent%20Practice.pdf

Variation supports Intelligent Practice

7 + 2 =	9 + 6 =	8 + 3 =	1 + 9 =
17 + 2 =	10 + 6 =	10 + 3 =	2 + 8 =
7 + 12 =	11 + 6 =	12 + 3 =	3 + 7 =
17 + 12 =	13 + 6 =	3 + 14 =	6 + 4 =





9 – 5 =	9 – 7 =
8 – 5 =	11 – 7 =
7 – 5 =	13 – 7 =
6 – 5 =	15 – 7 =

These exercises provide both the opportunity to practice calculation whilst at the same time thinking about the relationships within the mathematics.

Retrieved February 21, 2018 at https://www.ncetm.org.uk/public/files/22344481/Variation%20supports%20Intelligent%20Practice.pdf

Example: Practice with decimals & fractions

On the number line …

1. Draw a line (say 15 cm long) on a plain paper. Mark the left end 0 and the right end 1. By dividing the line between 0 and 1 into 10 parts, locate the decimal numbers 0.1, 0.2, …, 0.9 on the line. (Such a number line may also be provided.)
2. Given a list of fractions and/or decimals, e.g.

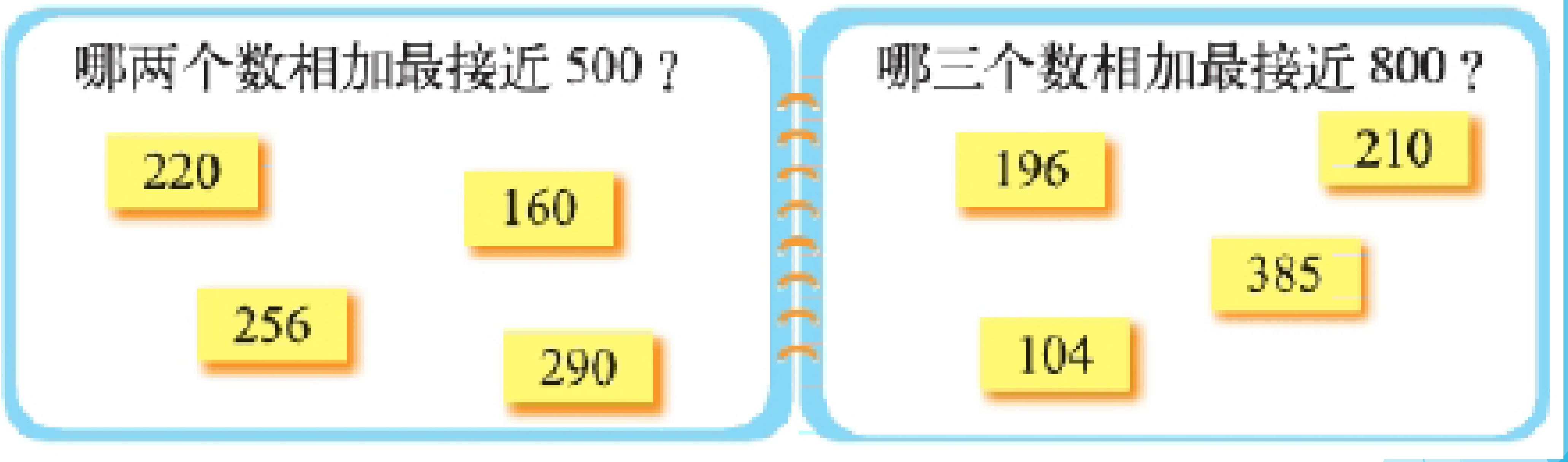
   0.5 , 1/4 , 2/5, 0.8 , 0.25 , 7/10 , 0.75 , 5/8

3. Estimate their location on the line.

(Adapted from Francome & Hewitt, p. 31)

Francome, T. & Hewitt, D. (2017). Practising mathematics: Developing the mathematician as well as mathematics. Derby, UK: Association of Teachers of Mathematics.

Example: Simple addition and …



Mathematics Book 3A published by Beijing Normal University 2014 (p. 18)

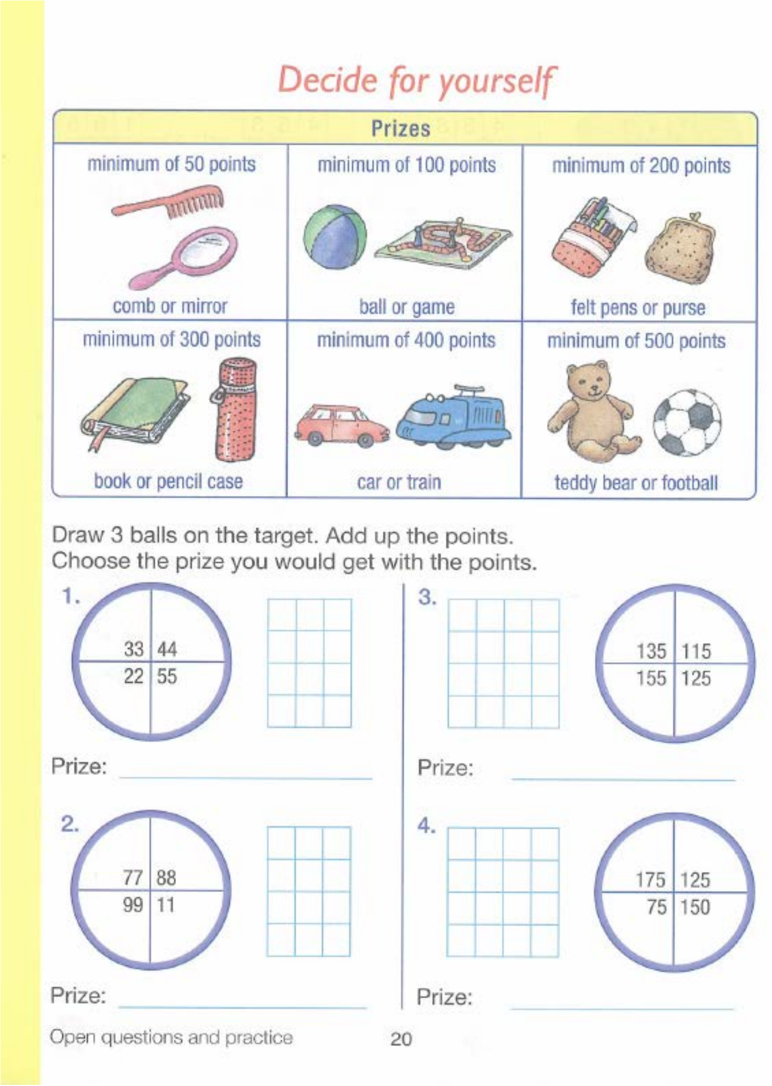
Example: Practice with multiplication and ...

Fill in a number (as large as possible) in the box.

1. 4 x □ is less than 17.
2. 6 x □ is less than 25.
3. □ x 5 is less than 43.
4. □ x 7 is less than 62.
5. □ x 8 is less than 38.
6. 9 x □ is less than 57.

Cf Mathematics Book 2B published by Beijing Normal University 2013 (p. 7)

Example: Simple addition and ...



Decide for yourself

Prizes		
minimum of 50 points comb or mirror	minimum of 100 points ball or game	minimum of 200 points felt pens or purse
minimum of 300 points book or pencil case	minimum of 400 points car or train	minimum of 500 points teddy bear or football

Draw 3 balls on the target. Add up the points.
Choose the prize you would get with the points.

1. 33 44 22 55

Prize: ____

2. 77 88 99 11

Prize: ____

3. 135 115 155 125

Prize: ____

4. 175 125 75 150

Prize: ____

Open questions and practice 20

Rikala, S. et al. (2006). Laskutaito (in English) 2B. Helsinki: WSOY Oppimateriaalit Oy.

Example: Practice with multiplication and …

1. 142857 x 1 =
2. 142857 x 2 =
3. 142857 x 3 =
4. 142857 x 4 =
5. 142857 x 5 =
6. 142857 x 6 =
7. 142857 x 7 =

Mathematics Book 4A published by Beijing Normal University 2014 (p. 38)

Example: Practice with basic operations

Target 24

The four numbers can be assigned as (1, 2, 3, 4), (1, 2, 3, 5), (1, 2, 3, 6), (4, 6, 6, 8), ...

1. Choose four (single-digit) numbers.
2. Use any of the four basic operations and brackets.
3. Use each number exactly once.
4. Write expressions to make 24.
5. Once you are done, try to carry on to get numbers greater than 20. How far can you go?

(Adapted from Francome & Hewitt, p. 24)

Francome, T. & Hewitt, D. (2017). Practising mathematics: Developing the mathematician as well as mathematics. Derby, UK: Association of Teachers of Mathematics.

Example: Practice with basic operations

One to Four

1. Use the numbers 1, 2, 3, and 4. You must use them all.
2. Use any of the four basic operations and brackets.
3. Write expressions to make every number from 1 to 20.
4. Once you are done, try to carry on to get numbers greater than 20. How far can you go?

(Adapted from Francome & Hewitt, p. 24)

Francome, T. & Hewitt, D. (2017). Practising mathematics: Developing the mathematician as well as mathematics. Derby, UK: Association of Teachers of Mathematics.

Example: Practice with basic operations

Four Fours

1. Use the numbers 4, 4, 4, and 4. You must use all four 4s.
2. Use any of the four basic operations and brackets.
3. Write expressions to make every number from 1 to 20.
4. Once you are done, try to carry on to get numbers greater than 20. How far can you go?

(Adapted from Francome & Hewitt, p. 24)

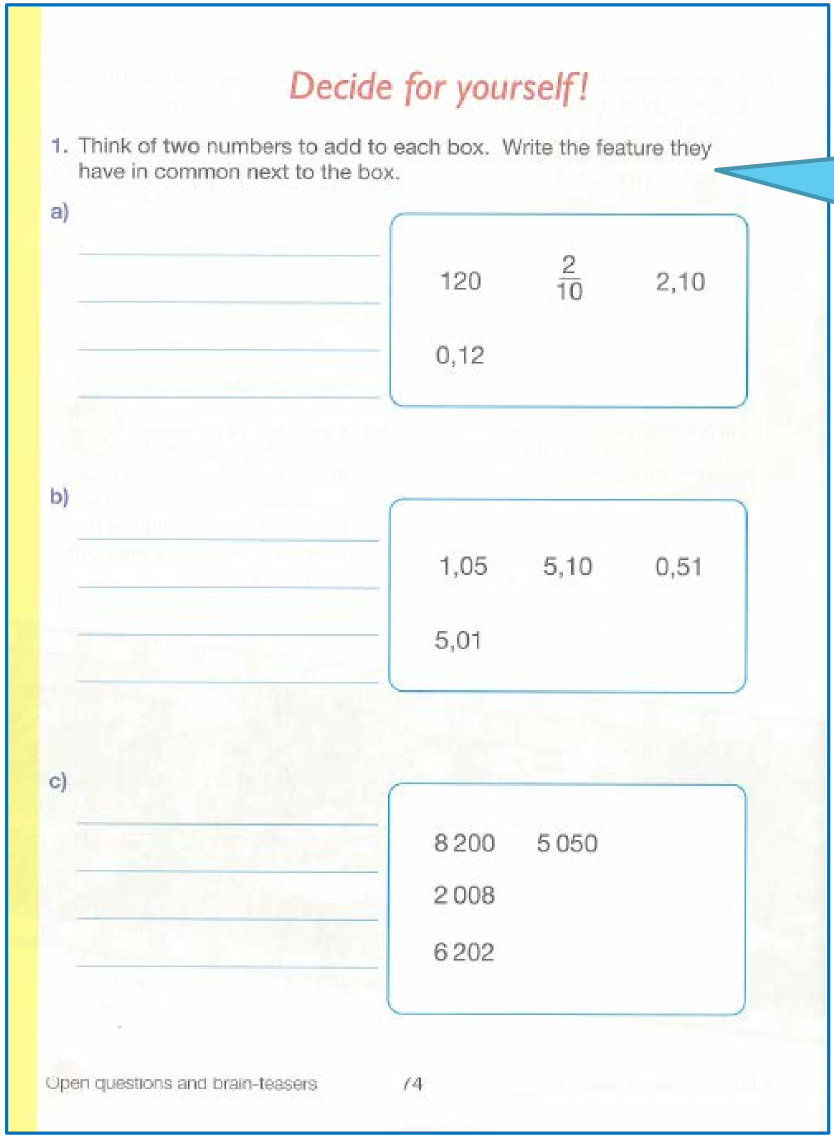
Francome, T. & Hewitt, D. (2017). Practising mathematics: Developing the mathematician as well as mathematics. Derby, UK: Association of Teachers of Mathematics.

Improve the **learning experience** in doing exercise

Exercise that …

- encourages students to think (talk)
- builds up concepts

Example: Basic ideas about decimals, fractions, ...



Decide for yourself!

1. Think of two numbers to add to each box. Write the feature they have in common next to the box.

a)

120	$\frac{2}{10}$	2,10
0,12		

b)

1,05	5,10	0,51
5,01		

c)

8 200	5 050
2 008	
6 202	

Open questions and brain-teasers 74

Think of two numbers to add to each box.
Write (Describe verbally) the feature they have in common next to the box.

Sintonen, A.-N. et al. (2006). Laskutaito (in English) 4A. Helsinki: WSOY Oppimateriaalit Oy.

Giving meanings to number sentences.

Describe a situation that fits the number sentence.

- $12 \times 4 = 48$

  Each pack contains 12 pencils, how many pencils are there in 4 packs?

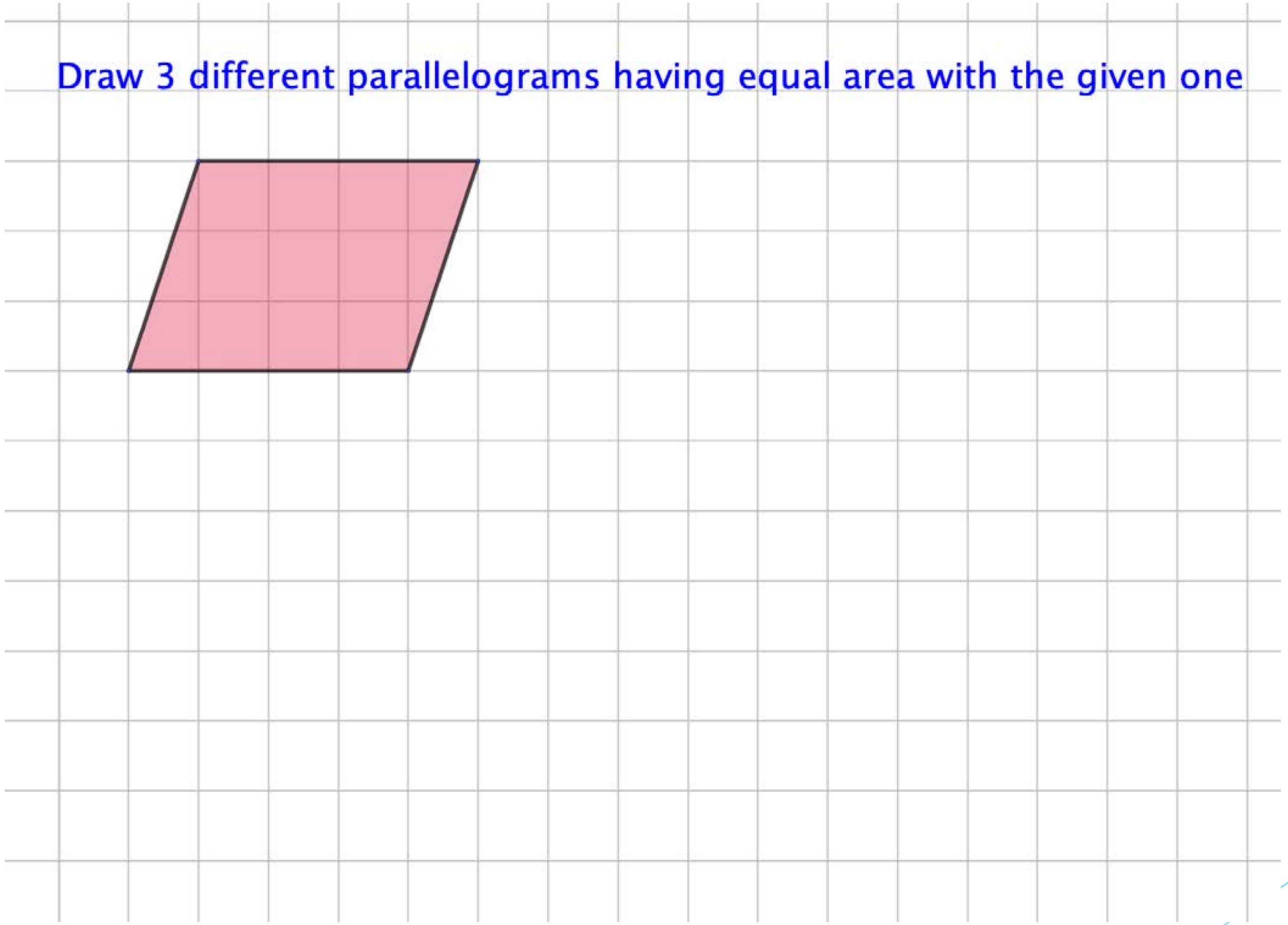
- $2 \div \frac{1}{3} = 2 \times 3 = 6$

  One third of a piece of bread is needed to make a sandwich, how many sandwiches could be made from 2 pieces of bread?

- $18 \div 4.5 = 4$

  Peter can walk 4.5 km each hour. How long will he take to walk 18 km?

Another example

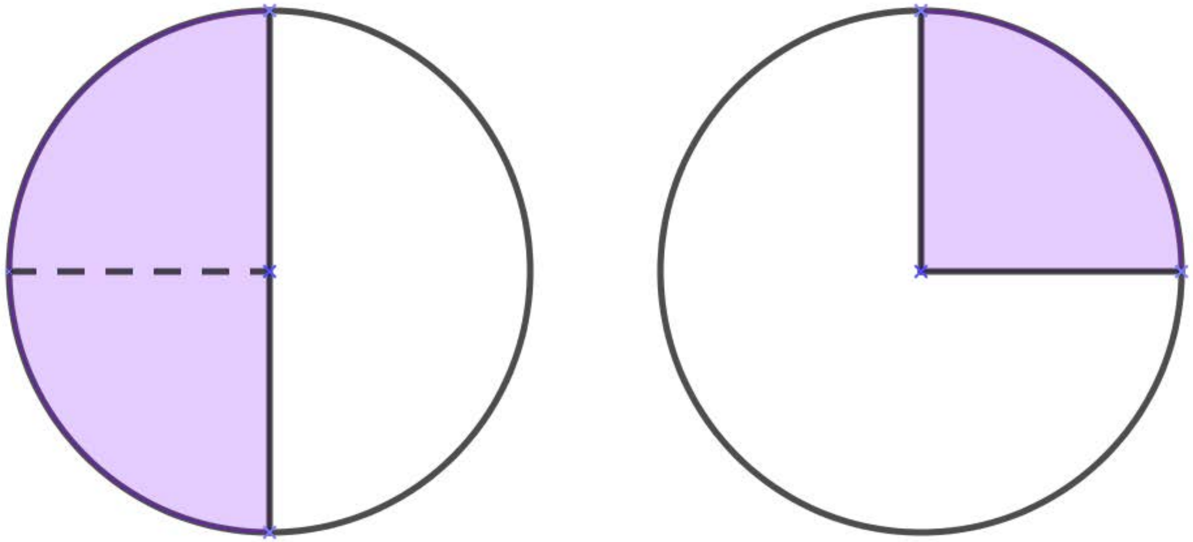


How will students respond and what further questions can be asked?

Good Exercise should …

- help students to show their understanding
  (the formula for area of parallelogram in this case)
- thus allow teacher / peers to provide support

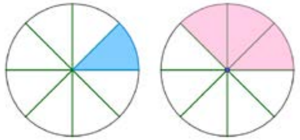
Example

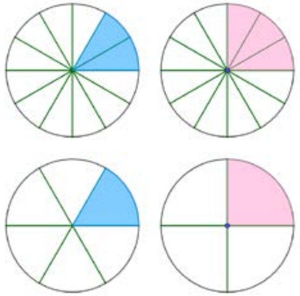


$$\frac{2}{4} + \frac{1}{4} = ?$$

Will the diagram help students to avoid some mistakes?

Examples



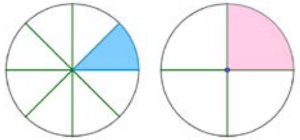


$\frac{1}{5} + \frac{2}{5} =$

$\frac{1}{8} + \frac{3}{8} =$

$\frac{1}{8} + \frac{1}{4} =$

$\frac{1}{6} + \frac{1}{4} =$



What is the advantage of questions with slight variation?

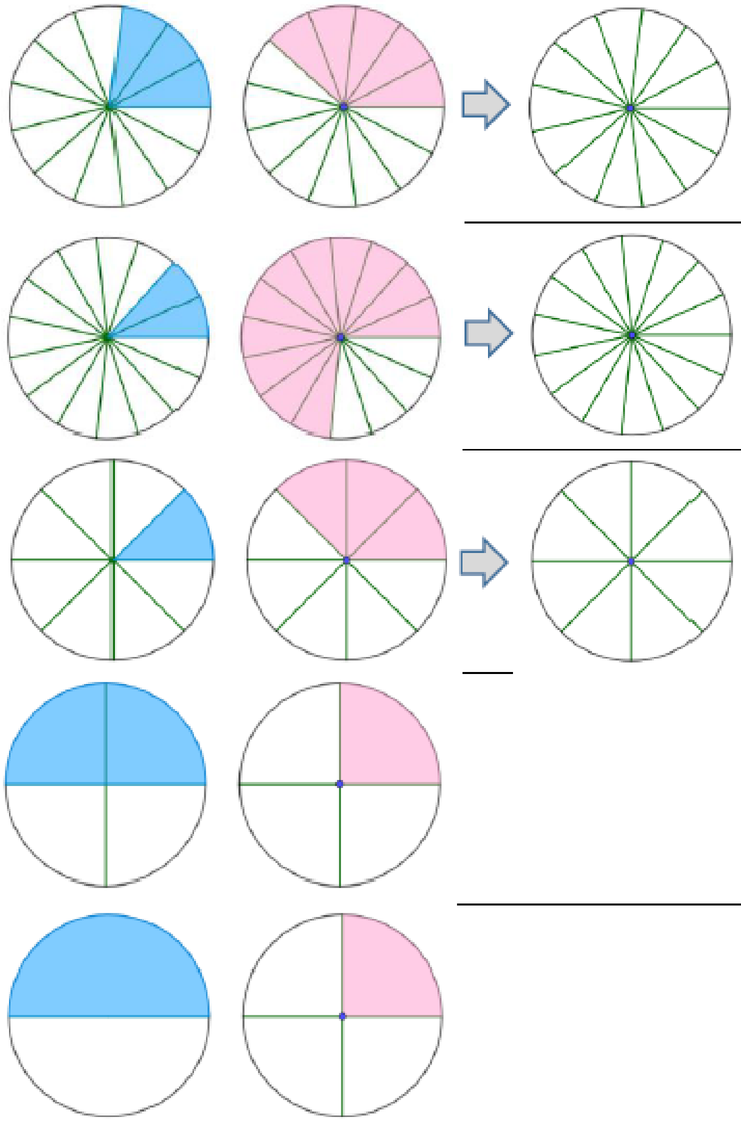
Examples

$$\frac{1}{5}+\frac{2}{5}=$$

What is the advantage of

Calculate the following (use diagrams to illustrate your working).

1. $\frac{3}{13}+\frac{5}{13}=$
2. $\frac{2}{15}+\frac{11}{15}=$
3. $\frac{1}{8}+\frac{3}{8}=$
4. $\frac{2}{4}+\frac{1}{4}=$
5. $\frac{1}{2}+\frac{1}{4}=$



Example: Practice with brackets and ...

Put in brackets to make the following calculations correct:

What next?

$$8 \times 5 - 4 + 12 \div 2 = 24$$
$$8 \times 5 - 4 + 12 \div 2 = 14$$
$$8 \times 5 - 4 + 12 \div 2 = 42$$
$$8 \times 5 - 4 + 12 \div 2 = -44$$
$$8 \times 5 - 4 + 12 \div 2 = 12$$
$$8 \times 5 - 4 + 12 \div 2 = 52$$
$$8 \times 5 - 4 + 12 \div 2 = 10$$
$$8 \times 5 - 4 + 12 \div 2 = 32$$
$$8 \times 5 - 4 + 12 \div 2 = -24$$

(Francome & Hewitt, p. 24)

Francome, T. & Hewitt, D. (2017). Practising mathematics: Developing the mathematician as well as mathematics. Derby, UK: Association of Teachers of Mathematics.

Example: Practice with brackets and …

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$8 \times 5 - 4 + 12 \div 2 = 14$

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$8 \times 5 - 4 + 12 \div 2 = -44$

$8 \times 5 - 4 + 12 \div 2 = 12$

$8 \times 5 - 4 + 12 \div 2 = 52$

$8 \times 5 - 4 + 12 \div 2 = 10$

$8 \times 5 - 4 + 12 \div 2 = 32$

$8 \times 5 - 4 + 12 \div 2 = -24$

Now use brackets to get different answers to any of those you have already got:

$8 \times 5 - 4 + 12 \div 2 =$

$8 \times 5 - 4 + 12 \div 2 =$

$8 \times 5 - 4 + 12 \div 2 =$

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(Francome & Hewitt, p. 24)

Francome, T. & Hewitt, T. (2017). Practising mathematics: Developing the mathematician as well as mathematics. Derby, UK: Association of Teachers of Mathematics.

Intelligent Practice

not only multiplication,
not simply quick / mental
calculation, but attention to
place values

2 × 3 =	6 × 7 =	9 × 8 =
2 × 30 =	6 × 70 =	9 × 80 =
2 × 300 =	6 × 700 =	9 × 800 =
20 × 3 =	60 × 7 =	90 × 8 =
200 × 3 =	600 × 7 =	900 × 8 =

Shanghai Textbook Grade 2 (aged 7/8)

"... rather than pupils repeating a mechanical activity, they are taken down a path where the thinking process is practised with increasing creativity."

"The arrangement of these tasks and exercises draw pupils' attention to patterns, structure and mathematical relationships, thus providing 'intelligent practice' and the opportunity to deepen conceptual understanding."

Source: http://www.mathshubs.org.uk/bespoke/april-2015/intelligent-practice/

Intelligent Practice

2 × 3 =	6 × 7 =	9 × 8 =
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Shanghai Textbook Grade 2 (aged 7/8)

"The practice that Chinese children engage in provides the opportunity to develop both procedural and conceptual fluency. Children are required to reason and make connections between calculations. The connections made improve their fluency."

(NCETM, 2015, p. 7)

NCETM (October 2015). *Calculation Guidance for Primary Schools*. https://www.ncetm.org.uk/public/files/25120980/NCETM+Calculation+Guidance+October+2015.pdf

Intelligent Practice

"The practice that Chinese children engage in provides the opportunity to develop both procedural and conceptual fluency. Children are required to reason and make connections between calculations. The connections made improve their fluency."

(NCETM, 2015, p. 7)

The above examples also show a common feature:
With the task at hand, students are working on many calculations which are not simply at the order of the teacher but naturally generated (partly by the students themselves) according to certain mathematical principles and/or out of their curiosity about certain unexpected results or patterns.

NCETM (October 2015). Calculation Guidance for Primary Schools. https://www.ncetm.org.uk/public/files/25120980/NCETM+Calculation+Guidance+October+2015.pdf