# Posing, Writing and Solving Word Problems through Mathematical Stories

利用數學繪本提出寫作和解決問題

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# The genre/linguistic form of WP

1 a Mum cuts a pizza equally into 8 pieces, and gives some pieces to Robert and Peter.

A pizza

Robert gets 1 piece, which is $\frac{1}{8}$ of the pizza.

Peter gets 2 pieces, which are $\frac{2}{8}$ of the pizza.

They get 3 pieces altogether, which are $\frac{3}{8}$ of the pizza.

To show this by addition: $\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$

b

A bottle of juice

There was $\frac{2}{5}$ of a bottle of juice at first.

Mum added $\frac{1}{5}$ of the bottle the juice she made.

Now there is $\frac{3}{5}$ of the bottle of juice.

To show this by addition: $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

1. Scenario
2. Information
3. Question

- The information is arbitrary in relation to the scenario.
- The use of tense and time is ambiguous.

Book 3D, Modern Primary Mathematics, 2nd Edition

When solving word problems, children—

> frequently choose an operation without making sense of the choice. . . . Knowing why an operation is an appropriate choice for a solution strategy is an important part of establishing a robust understanding of mathematics
>
> (Otto et al. 2011, p. 15)

# Why storytelling?

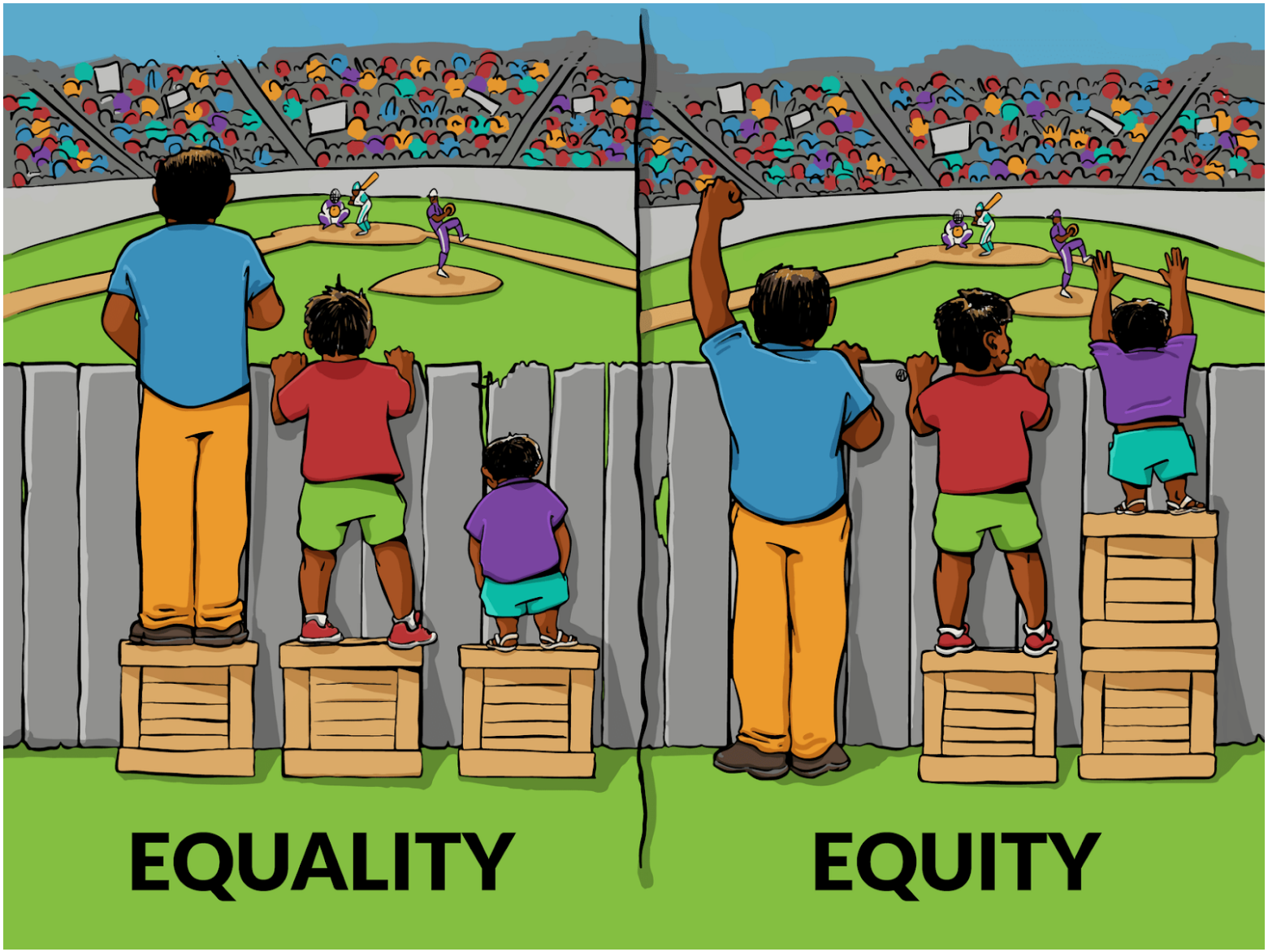
To pose challenging mathematical tasks for ALL students and making them accessible to ALL levels of language development

(Ramirez & Celedón-Pattichis, 2012)

To develop instruction that addresses in INEQUITIES that often exist for emergent bilingual children by making challenging word problems accessible even before or as they are mastering the basic skills

(Turner & Celedón-Pattichis, 2011)

# Equity



# Connecting mathematics content to language

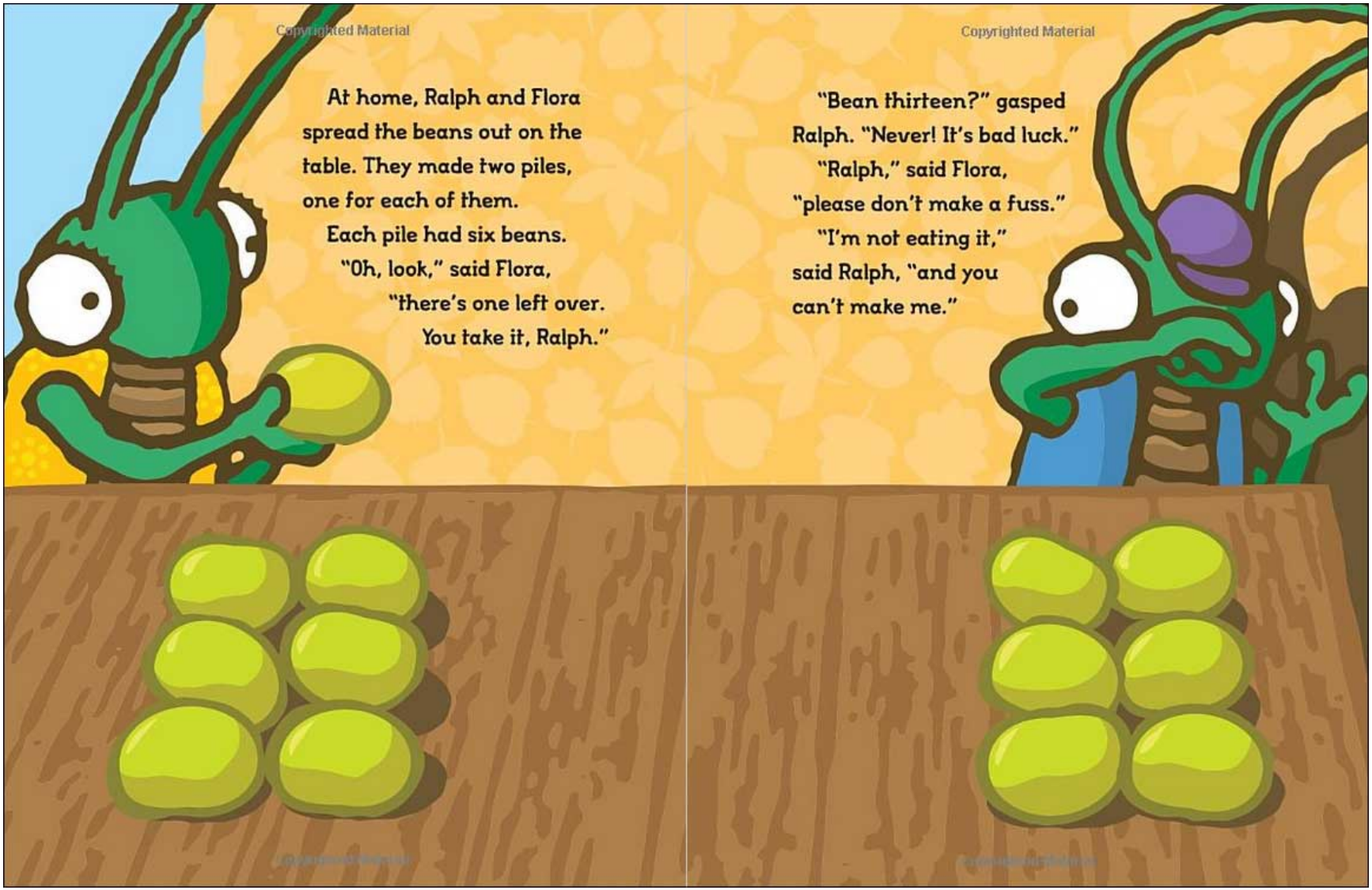
- Both linguistic and numerical complexity contribute to the difficulty in solving arithmetic WPs.
- Students from minority cultural/linguistic backgrounds tends to have greater difficulties with WPs, they are likely to find the relationship between the quasi-real world of the problem and their own experiences of the world difficult to negotiate (Barwell, 2005).
- Learning mathematical language in bilingual mathematics classroom should go beyond vocabulary and technical usage, students should participate in a community where they learn to mathematize situations and to use language to communicate about the situations (Moschkovich,1996).

# Bean Thirteen

Stories appear to students who may be conversationally proficient, while their proficiency in using mathematically English/Chinese are still developing…

# Bean Thirteen

Culturally Responsive Teaching acknowledges and honours students' cultural backgrounds and ways of learning (Gay, 2000) https://www.nationalgeographic.com/news/2013/9/130913-friday-luck-lucky-superstition-13/



# Storytelling

Simply decoding words/ extracting arithmetic operations is not enough. A question makes sense only to the extent that you understand the context of the given problem and to determine what mathematics is needed to solve the problem.

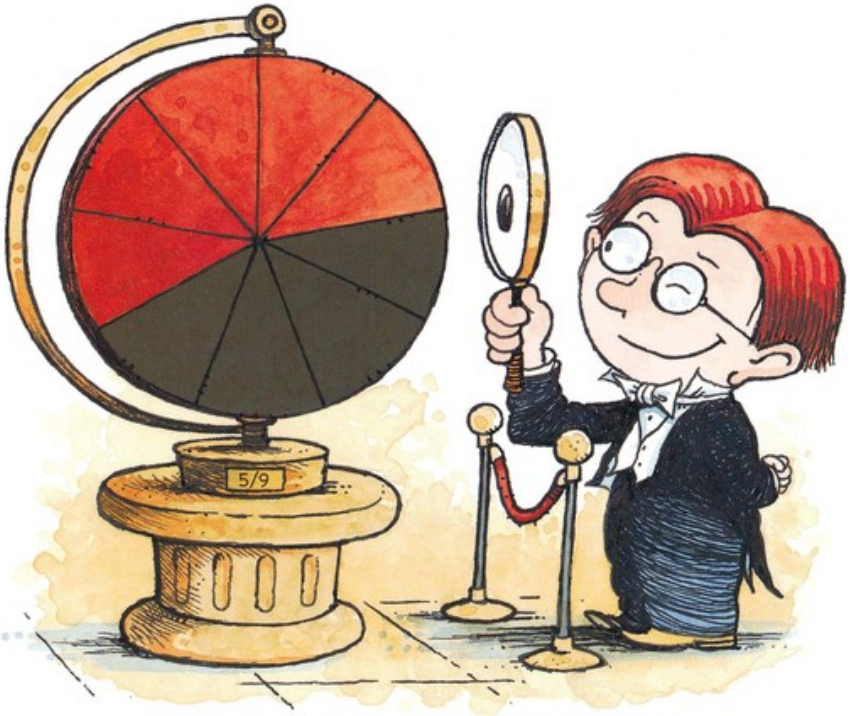
- Address the different meanings possible when participants come from diverse linguistic and cultural backgrounds, "*What do you know about the number 13?*".
- Situational storytelling provides a semantic structure that engages students in understanding, mathematizing, analysing and communicating in a meaningful context, "*How was their way of sharing the beans different from what we call equal shares*?"
- Storytelling supports the conceptual understanding of mathematical problems and structure (scenario, information and question). "*How did you divide the beans? What number sentence would show the way the beans are grouped now?*"

# Fractions in Disguise

So when a brand-new 5/9 went up for auction, you know I was first in line to buy it. The 5/9 is a thing of beauty. When you first look at it, it looks like a 1/2, but the more you look, the more you realize it's just a little bit more.

The room was filled with the regular customers: Baron von Mathematik, Madame de Géométrique, and the mysterious Dr. Brok, a former university professor rumored to have been fired for the illegal possession of a 4/0.

I bid 1/2 of a million dollars. Madame de Géométrique bid 3/4 of a million. Baron von Mathematik bid 7/8 of a million. Our bids were clearly approaching one million dollars. Would we ever reach it?



# Problems in Context

- The problem-solving context is much more convincing when it occurs naturally as part of the story, and teachers can take story characters out of the books into the classroom and to create mathematical problem/activity.
- Students can relate to the word problems in the story (or problems of characters might encounter) with their own experiences, and use their knowledge/experiences in solving word problems.
- Posing problems and proving activities within a meaningful context through storybooks is a way to make mathematics relevant and helps students to link their knowledge to different situations.
- Multiple representations in stories support comprehension in problem solving, understanding textual information and data.

# The Lion's Share


The concepts of halving, doubling and fractions in the same story.

- Classroom discussion can focus on exploring fractions (equal parts) and fraction language (such as halves, fourths, eighths, sixteenths) to help students in building language that let them participate in class.
- More problems could be created by comparing and ordering fractions.
- Personal experiences can be used to make the underlying mathematical structure meaningful and to interpret the scenario of the problems.

https://www.youtube.com/watch?v=kI9kjPvKC0E&t=3s

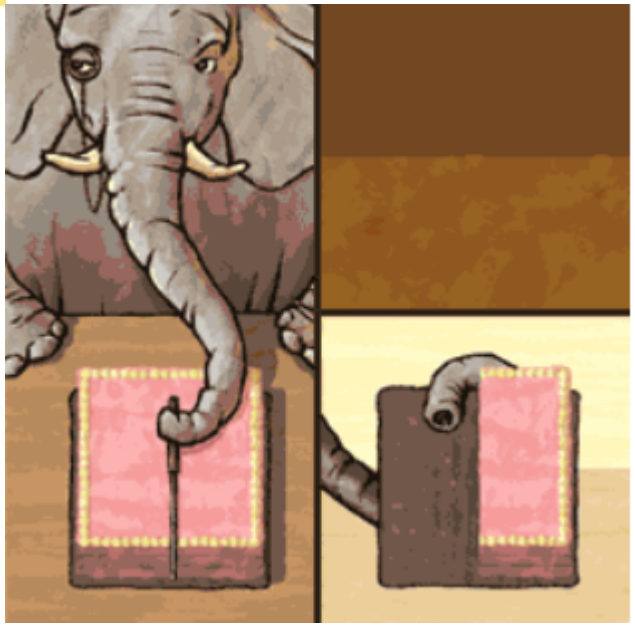
# The Lion's Share

The lion invited the ant to join him for a special dinner.

# The Lion's Share: A tale of halving cake and eating it, too

After dinner, a large cake was brought out for dessert. The lion passed it to the elephant.

Problem Posing:
Students would need to think about "*What is a whole? What is a half?*"

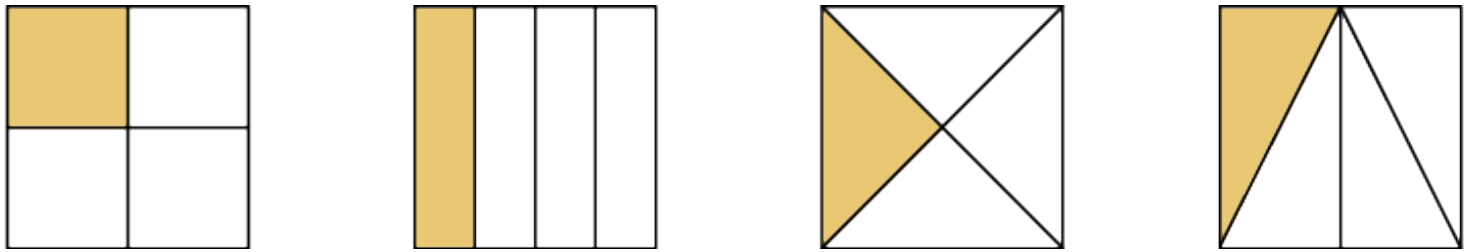


He cut the cake in half and …

passed the rest to the hippo, and said "but if he's taking half, I'm taking half of what's left."

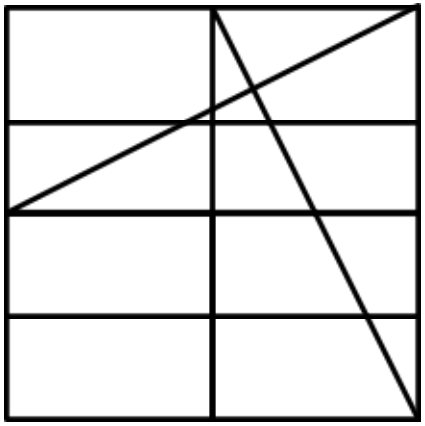
# One quarter of the cake

Problem-Posing: *What is the same about each part? What is different?*
[Talk about HOW they are the same and different, and which one seems more fair.]



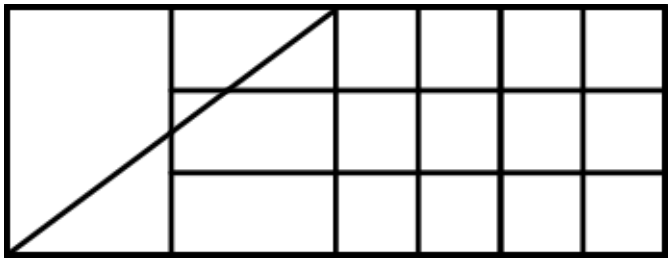
Note that: Students easily confuse dividing into fourths and dividing by one-fourth.

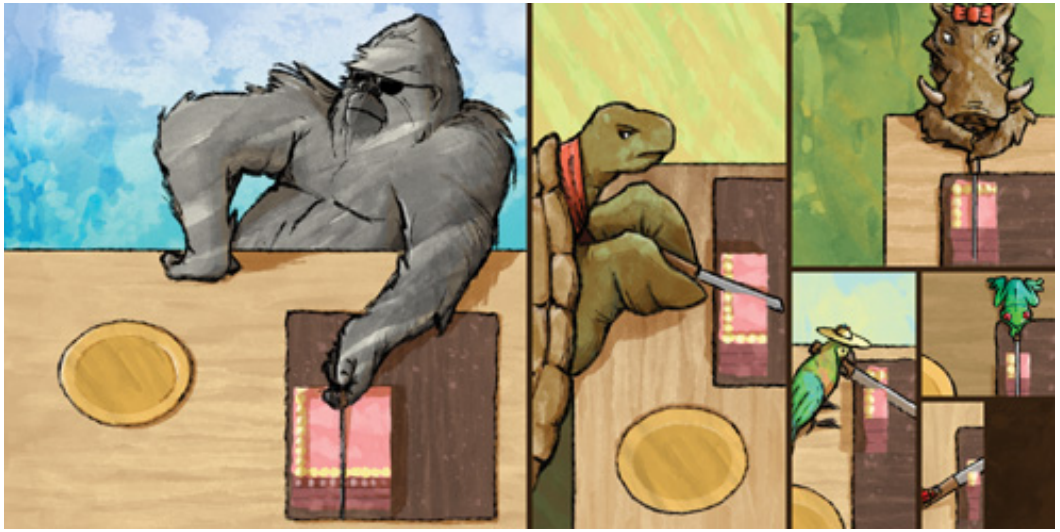
EXPLORING EQUAL PARTS: *Equal or unequal parts?*



Problem-Posing: Give students a piece of paper and ask them to think about how they could fold the paper to create eight (10, 12, etc.) equal parts.

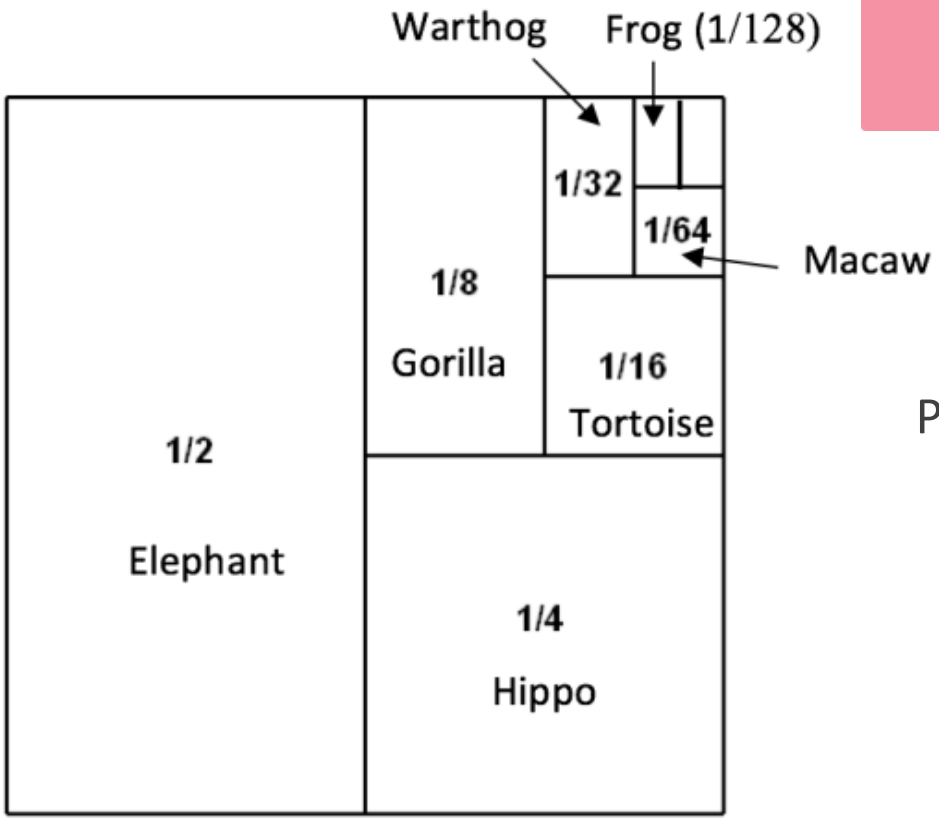
*Equal parts?*





This continued around the table ... as each animal took half of the remaining cake and passed the rest on. Until...

# Compare Unit Fractions

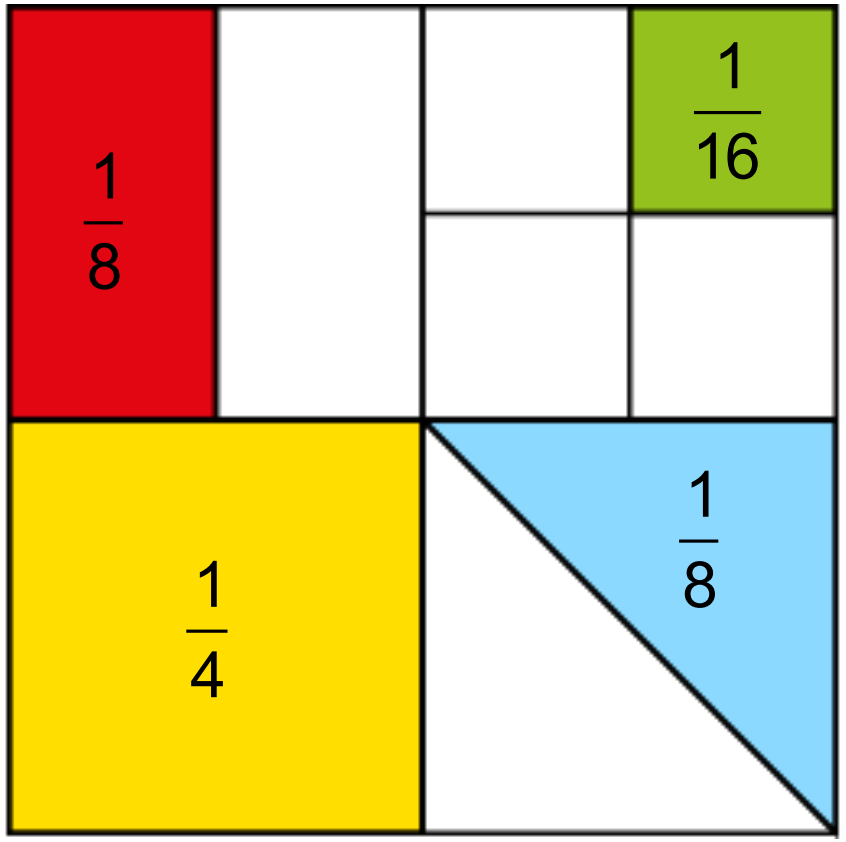




Problem-Posing: *Why didn't the ant have as much as the elephant?*

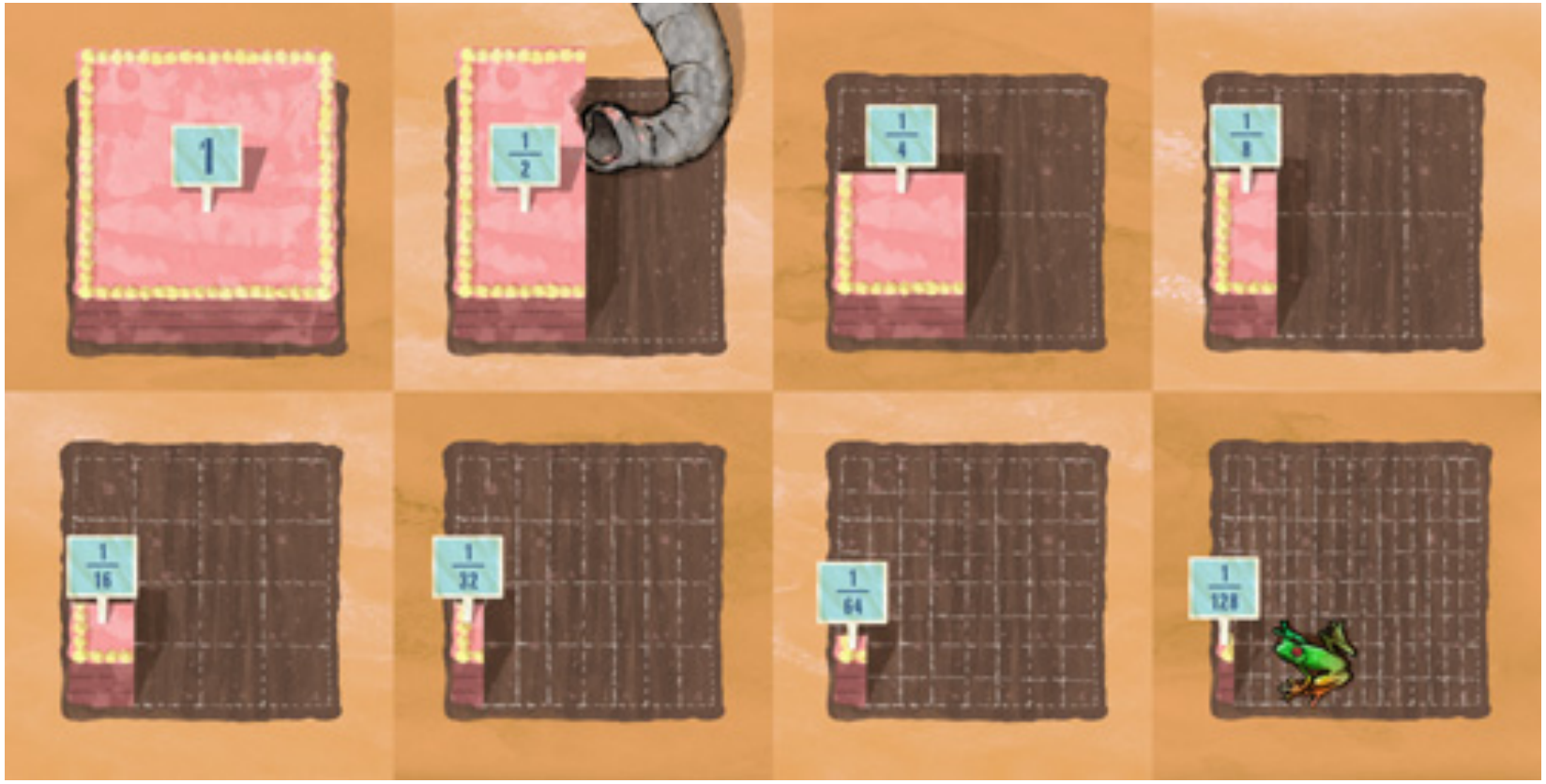
https://mathbookmagic.com/2017/09/19/the-lions-share-a-story-to-share-on-mathstorytellingday/

# Compare Unit Fractions

Problem-Posing : *What fraction of the whole square is green/red/light blue/yellow?*



# Compare Unit Fractions

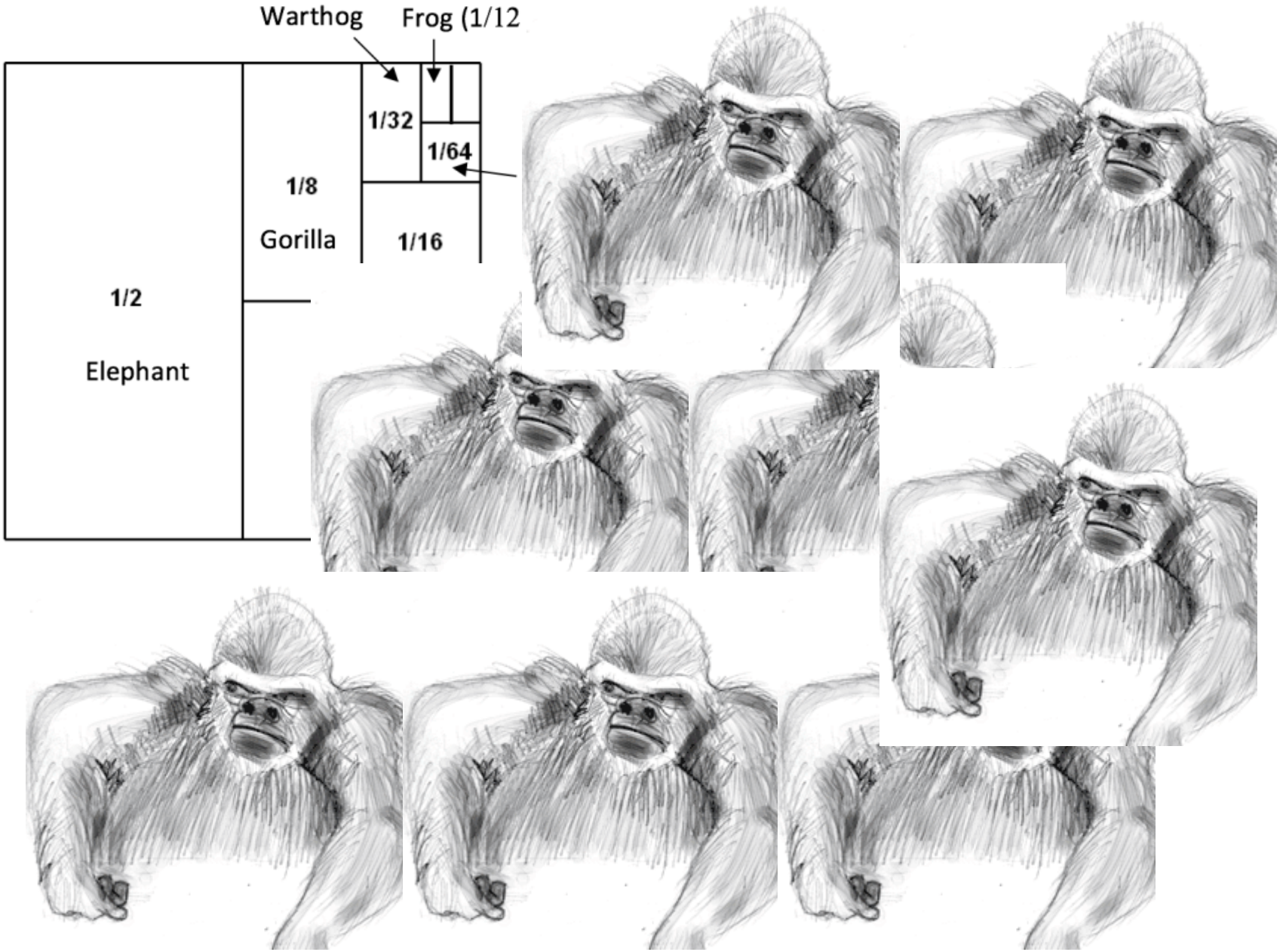



Problem-Posing: *Why didn't the animals split the cake equally?* [TALK about the fairness and unfairness. And how you would feel if you are the ant.]

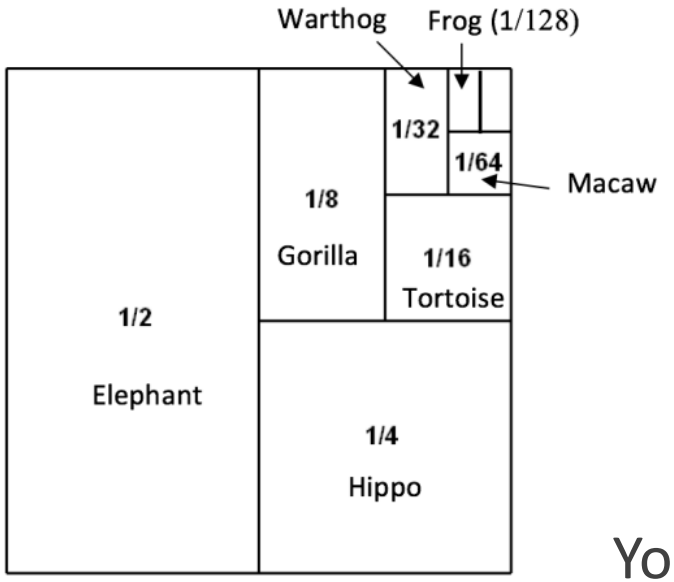
# Exploring Pattern

| Part | Part as a fraction of the whole | Number of equal parts in the whole | Whole |
|---|---|---|---|
| ☐ | $\frac{1}{3}$ | | |
| ☐ | $\frac{1}{4}$ | | |
| ☐ | $\frac{1}{5}$ | | |
| ☐ | $\frac{1}{6}$ | | |
| ☐ | $\frac{1}{7}$ | | |
| ☐ | $\frac{1}{8}$ | | |

# Gorillas …



# Counting fractional parts (the denominator)




You can count, 1-eighth, 2-eighths, 3-eighths, . . . , up to 8-eighths to total one. This language reinforces the fact that the numerator tells ***how many parts*** and that the denominator tells ***the size of the parts***.

Consider the difference in a students hearing

## 1 over 4 times 2

Saying 1 over 4 communicates to students that the fraction is two whole numbers.

## 1-fourth of 2

Saying fractions using "*ths*" connects to students that the fraction represents a single value. This also develop a better understanding of division of fractions, e.g. $4 \div 1/8$

## *How many eighths are in* 4?

# Language counts

Partition the 4 wholes into eighths and then iterate (emphasizing the *eighths* as you read), 1-eighth, 2-eighths, . . . , 32-eighths.

And consider that there are 8-eighths in a whole,

or 32-eighths in 4 wholes.

And you see why we multiply by the denominator in the standard algorithm for division of fractions and why it works. Using "*ths*" for fractions and emphasizing iterating can potentially help more students understand division of fractions, as well as other operations with rational numbers.

Van de Walle, Karp, and Bay-Williams (2013)

# Language counts

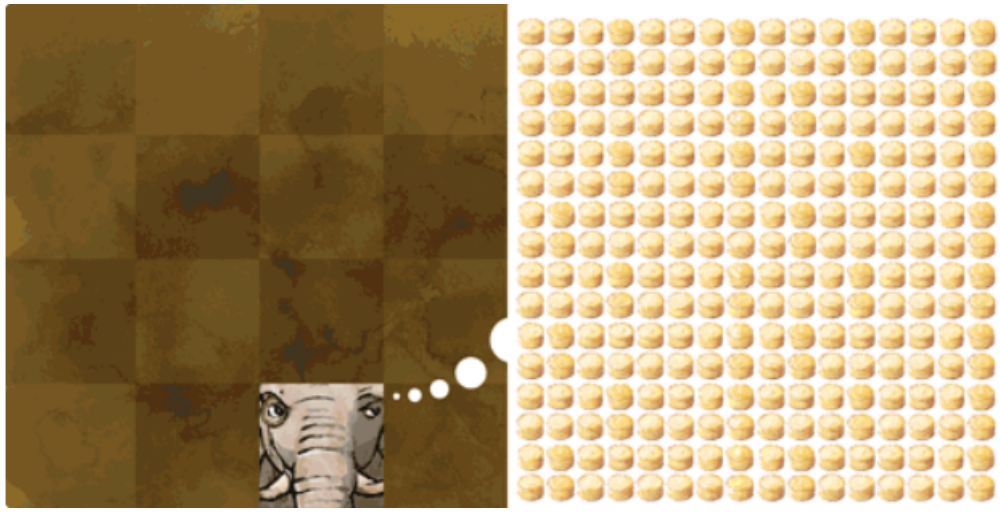
Using precise language that focuses on the meaning of fractions and on equivalence, can play an important role in helping students understand fractions (including operations and representations and use them in efficient and accurate ways.

Bay-Williams, 2013

The ant ...

# The Lion's Share

Part 2: Each animal ended up baking for the lion …1, 2, 4, 8, 16, 32, 64, 128, 256. "256 cakes! That's a lot of cake."



# Posing High Level Thinking Questions

## Open-ended with more than one right answer

1. *How could you cut the cake and share among the animals, equally? Use diagram/fraction pieces to show your thinking.* [TALK about how they are the same and different, and which one seems more fair.]
2. *What if the lion's wife also want to have a piece of the cake? How could you do this?*
3. *What if the cake the is not a square? But a trapezoid? A hexagon?* Show the different ways of sharing the cake equally.

## Types of problems

- Character-posed problems
- Teacher-posed problems
- Student-posed problems, which explore students' natural disposition to wonder and ask questions.

# Problem-Writing

Generating and solving new problems from previously solved problems help students to deepen mathematical understanding. Students can

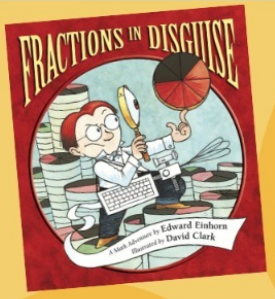
1. Revise/rewrite the given problems to make it more comprehensible or in their own context, e.g. different food items. "*How can I change this problem?*";
2. Change or add new problem conditions, quantities and words that represented those quantities;
3. Extend the problems in different ways, e.g. transforming into other fraction problems while maintaining the scenario.

# Problem-Writing

Having students to create their own mathematical stories and pictures

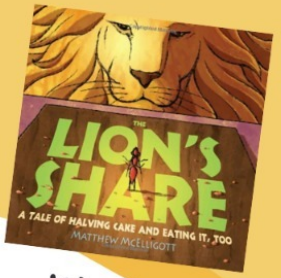
- provides teachers with important information about their knowledge/understanding of operations and quantities;
- develops students' expertise in interpreting problems and carrying out appropriate computation to solve them;
- allows students to revise their understanding of mathematics concepts and clarify misconceptions.
- offers opportunities for teachers to facilitate classrooms discussions, e.g. *"what did you know?"*, *"what were you trying to find out in your problem*?"

# HKU Maths Book Club



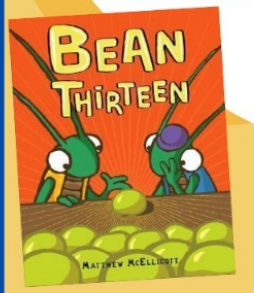

## P4 Expanding and Reducing Fractions

10 March 2020 (Tuesday)
2:30pm - 3:30pm
**Fractions in Disguise** by Edward Einhorn




## P2 Division

3 March 2020 (Tuesday)
2:30pm - 3:30pm
**Bean Thirteen** by Matthew McElligott




## P3 Fraction Concepts

Date & Time (TBC)
**The Lion's Share** by Matthew McElligott

LEARNING MATHEMATICS THROUGH

# Storytelling

QTN-T Project: Supporting the Learning and Teaching of Mathematics for Non-Chinese Speaking (NCS) Students in Primary Schools (2019-2020)

**MATHEMATICS AND ENGLISH TEACHERS ARE WELCOME**

In the meetings, learning and teaching materials will be discussed and designed for online and/or face-to-face classrooms.

Participating teachers are encouraged to read the selected story before the meeting. Relevant online resources will be provided beforehand.

# References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- Bay-Williams, J. M. (2013). 5 language substitutions when teaching fractions. *Mathematics Teaching in the Middle School*. NCTM. 19(2), 68-69.
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- Ramirez, N., & Celedón-Pattichis, S. (2012). Understanding second language development and implications for the mathematics classroom. In S. Celedón-Pattichis & N. Ramirez (Eds.), (pp. 19-37). Reston, VA: National Council of Teachers of Mathematics.
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- Van de Walle, John A., Karen S. Karp, & Jennifer M. Bay-Williams. 2013. *Elementary and Middle School Mathematics Methods: Teaching Developmentally, Professional Development*. New York: Allyn & Bacon.
- Some of the slides are adapted from www.ncetm.org.uk/masterypd